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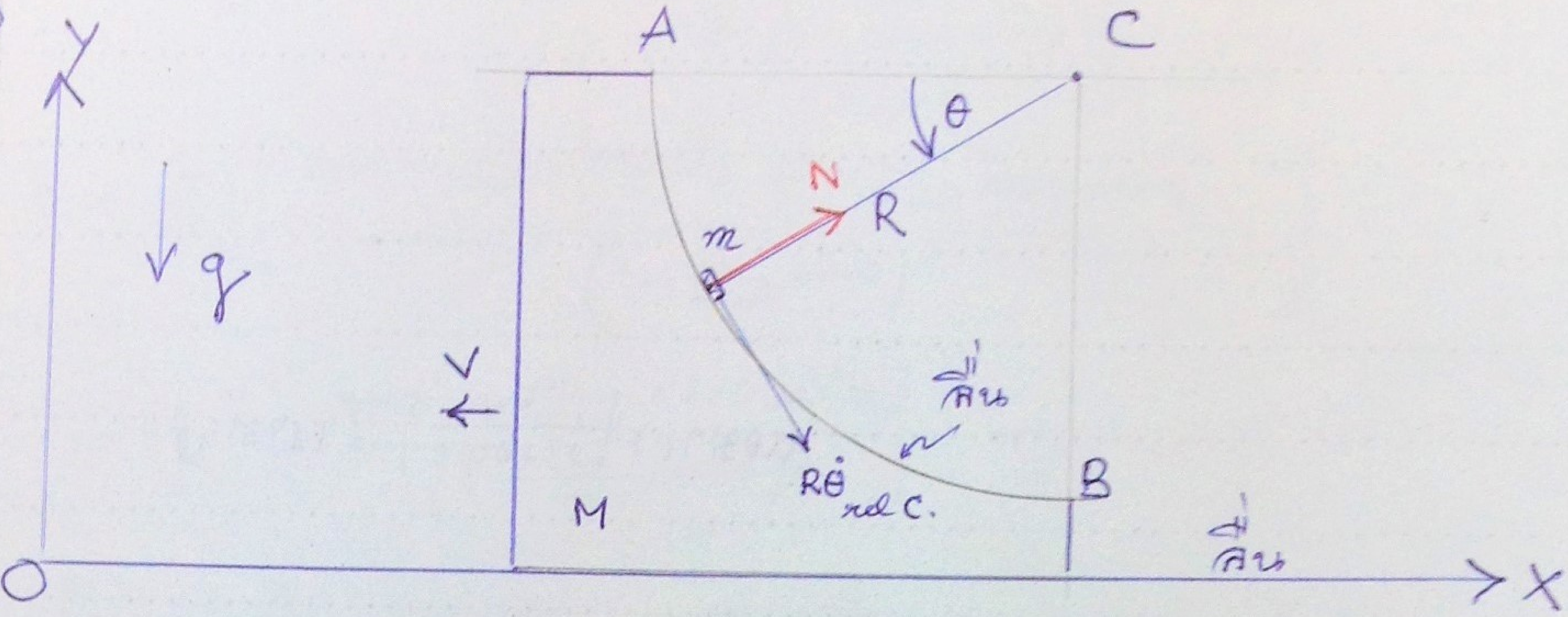
1.



ก) อนุรักษ์โมเมนตัมในแนวแกน X : $m(R\dot{\theta}\sin\theta - V) = MV$ —— ①

$$\therefore V = \frac{m}{m+M} R\dot{\theta}\sin\theta \quad \text{—— (1.1)}$$

อนุรักษ์พลังงาน : $\frac{1}{2}m(R\dot{\theta}\sin\theta - V)^2 + \frac{1}{2}m(R\dot{\theta}\cos\theta)^2 + \frac{1}{2}MV^2 = mgR\sin\theta$ —— ②

$$\frac{1}{2}mR^2\dot{\theta}^2(\sin^2\theta + \cos^2\theta) - mR\dot{\theta}\sin\theta\, V + \frac{1}{2}mV^2 + \frac{1}{2}MV^2 = mgR\sin\theta$$

$$\frac{1}{2}mR^2\dot{\theta}^2 - mR\dot{\theta}\sin\theta\, V + \frac{1}{2}(m+M)V^2 = mgR\sin\theta \quad \text{—— (2.1)}$$

(1.1) & (2.1) ได้ $\frac{1}{2}mR^2\dot{\theta}^2 - mR\dot{\theta}\sin\theta \cdot \frac{m}{m+M}R\dot{\theta}\sin\theta + \frac{1}{2}(m+M)\frac{m^2}{(m+M)^2}R^2\dot{\theta}^2\sin^2\theta = mgR\sin\theta$

$$\frac{1}{2}mR^2\dot{\theta}^2 - \frac{1}{2}\frac{m^2}{m+M}(R\dot{\theta})^2\sin^2\theta = mgR\sin\theta$$

$$\dot{\theta}^2 = \frac{\left(\frac{2g}{R}\right)\sin\theta}{\left(1 - \frac{m}{m+M}\sin^2\theta\right)}$$

ข) ใช้ Second law : $m\frac{d}{dt}(R\dot{\theta}\sin\theta - V) = N\cos\theta$ —— ③

$$m\frac{d}{dt}(R\dot{\theta}\cos\theta) = mg - N\sin\theta \quad \text{—— ④}$$

$$M\frac{d}{dt}V = N\cos\theta \quad \text{—— ⑤}$$

⑤ & (1.1) $N\cos\theta = M\frac{d}{dt}\left(\frac{m}{m+M}R\dot{\theta}\sin\theta\right)$

$$= \frac{mMR}{m+M}\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right)$$

$$N = \frac{mMR}{m+M}\left(\tan\theta\,\ddot{\theta} + \frac{\left(\frac{2g}{R}\right)\sin\theta}{\left(1 - \frac{m}{m+M}\sin^2\theta\right)}\right) \quad \text{—— ⑥}$$

1. ([illegible]) dt. 15.10.65

$$\frac{d}{dt}\dot{\theta}^2 \text{ [illegible] } 2\dot{\theta}\ddot{\theta} = \left(\frac{2g}{R}\right)\left\{\frac{\cos\theta\,\dot{\theta}\left(1-\frac{m}{m+M}\sin^2\theta\right)+(\sin\theta)\,2\frac{m}{m+M}\sin\theta\cos\theta\,\dot{\theta}}{\left(1-\frac{m}{m+M}\sin^2\theta\right)^2}\right\}$$

$$\ddot{\theta} = \left(\frac{g}{R}\right)\left[\frac{\cos\theta-\frac{m}{m+M}\cos\theta\sin^2\theta+2\frac{m}{m+M}\cos\theta\sin^2\theta}{\left(1-\frac{m}{m+M}\sin^2\theta\right)^2}\right]$$

$$= \left(\frac{g}{R}\right)\left[\frac{\cos\theta+\frac{m}{m+M}\cos\theta\sin^2\theta}{\left(1-\frac{m}{m+M}\sin^2\theta\right)^2}\right] \quad \text{——— } (7)$$

[illegible] ([illegible])

(6) & (7):

$$N = \frac{mMR}{m+M}\left[\frac{\sin\theta+\frac{m}{m+M}\sin^3\theta}{\left(1-\frac{m}{m+M}\sin^2\theta\right)^2}+\frac{2\sin\theta}{\left(1-\frac{m}{m+M}\sin^2\theta\right)}\right]\left(\frac{g}{R}\right)$$

$$N = \frac{mMg}{m+M}\left[\frac{\sin\theta+\frac{m}{m+M}\sin^3\theta}{1-\frac{m}{m+M}\sin^2\theta}+2\sin\theta\right]\frac{1}{\left(1-\frac{m}{m+M}\sin^2\theta\right)}$$

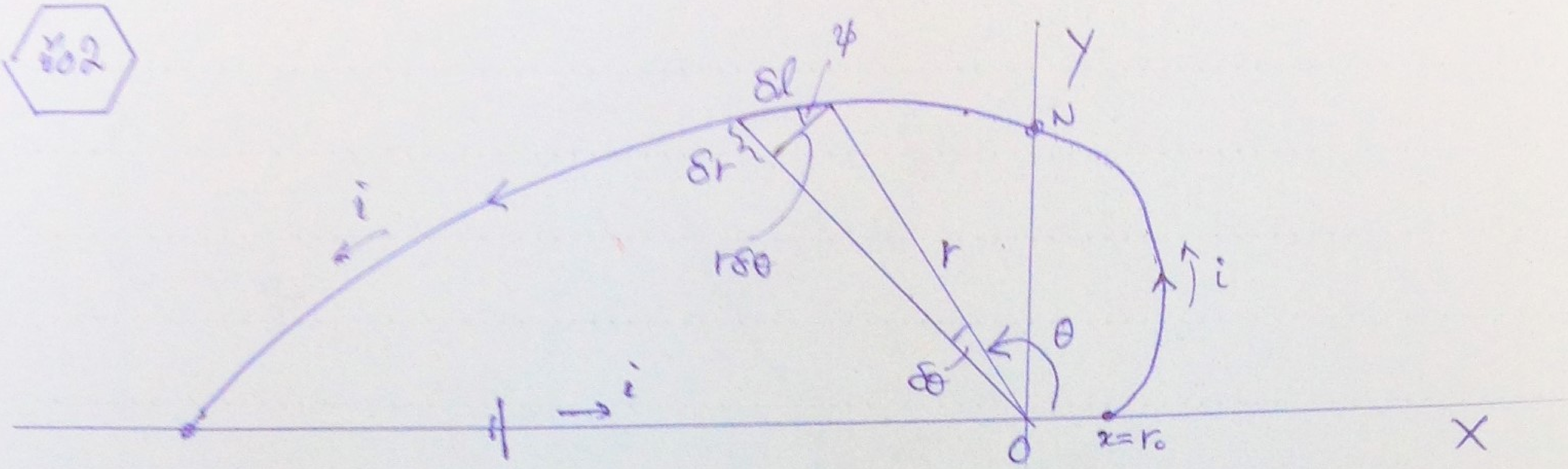
$$= \frac{mMg}{m+M}\left[\frac{\sin\theta+\frac{m}{m+M}\sin^3\theta+2\sin\theta-2\frac{m}{m+M}\sin^3\theta}{\left(1-\frac{m}{m+M}\sin^2\theta\right)}\right]$$

$$= \frac{mMg}{m+M}\left[\frac{3\sin\theta-\frac{m}{m+M}\sin^3\theta}{1-\frac{m}{m+M}\sin^2\theta}\right]$$

$$N = \left(\frac{mMg}{m+M}\sin\theta\right)\left[\frac{3-\frac{m}{m+M}\sin^2\theta}{1-\frac{m}{m+M}\sin^2\theta}\right] \quad \text{——— } (8)$$

[illegible] ([illegible])

ข้อ2



ก) $$\delta B_{\text{ที่ O}} = \frac{\mu_0}{4\pi}\frac{i\,\delta l \sin(90^\circ+\psi)}{r^2} \quad \text{ชี้ออกมา}$$

$$= \frac{\mu_0 i}{4\pi}\frac{\delta l \cos\psi}{r^2} \;, \quad \cos\psi = \frac{r\delta\theta}{\delta l} \;,$$

$$= \frac{\mu_0 i}{4\pi}\frac{\delta\theta}{r} \;, \quad r = r_0 e^{\alpha\theta}$$

$$= \frac{\mu_0 i}{4\pi r_0} e^{-\alpha\theta}\delta\theta$$

$$B_0 = \frac{\mu_0 i}{4\pi r_0}\int_{\theta=0}^{\pi} e^{-\alpha\theta} d\theta = -\frac{\mu_0 i}{4\pi\alpha r_0}\left[e^{-\alpha\theta}\right]_{\theta=0}^{\pi}$$

$$= \frac{\mu_0 i}{4\pi\alpha r_0}\left(1 - e^{-\alpha\pi}\right)$$

ตอบ

ข) เมื่อลัดวงจรจาก O ไป N , ตัดเส้นเดิมขวาแกน Y ทิ้ง,

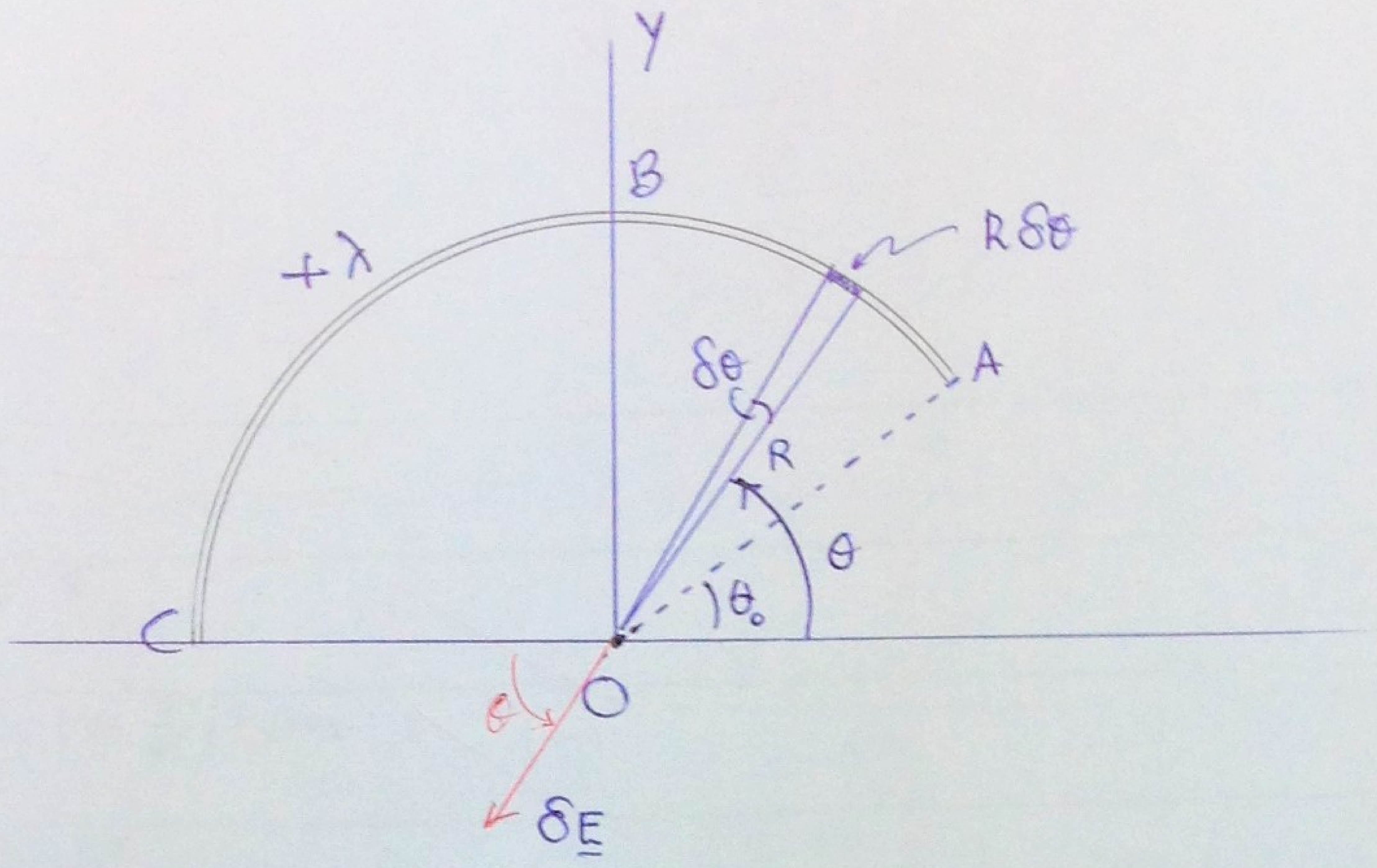
$$B_0 = \frac{\mu_0 i}{4\pi r_0}\int_{\theta=\frac{\pi}{2}}^{\pi} e^{-\alpha\theta} d\theta = -\frac{\mu_0 i}{4\pi\alpha r_0}\left[e^{-\alpha\theta}\right]_{\theta=\frac{\pi}{2}}^{\pi}$$

$$= -\frac{\mu_0 i}{4\pi\alpha r_0}\left(e^{-\alpha\pi} - e^{-\alpha\frac{\pi}{2}}\right) = \frac{\mu_0 i}{4\pi\alpha r_0}\left(e^{-\alpha\frac{\pi}{2}} - e^{-\alpha\pi}\right)$$

ตอบ

st. 15.10.65

3.



ก $$\delta E_x = \frac{-\lambda R\delta\theta}{4\pi\varepsilon_0 R^2}\cos\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta\,\delta\theta$$

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_{\theta=\theta_0}^{\pi}\cos\theta\, d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}(\sin\pi - \sin\theta_0)$$

$$= \frac{\lambda\sin\theta_0}{4\pi\varepsilon_0 R}$$ ตอบ

ข $$\delta E_y = -\frac{\lambda R\delta\theta}{4\pi\varepsilon_0 R^2}\sin\theta$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 R}\Big[\cos\theta\Big]_{\theta=\theta_0}^{\pi} = -\frac{\lambda}{4\pi\varepsilon_0 R}(1+\cos\theta_0)$$ ตอบ

ค $$|\underline{E}|^2 = E_x^2 + E_y^2 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\left[\sin^2\theta_0 + 1 + 2\cos\theta_0 + \cos^2\theta_0\right]$$

$$= \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2(2+2\cos\theta_0)$$

$$|\underline{E}| = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)(\sqrt{2})(1+\cos\theta_0)^{\frac{1}{2}}$$ ตอบ

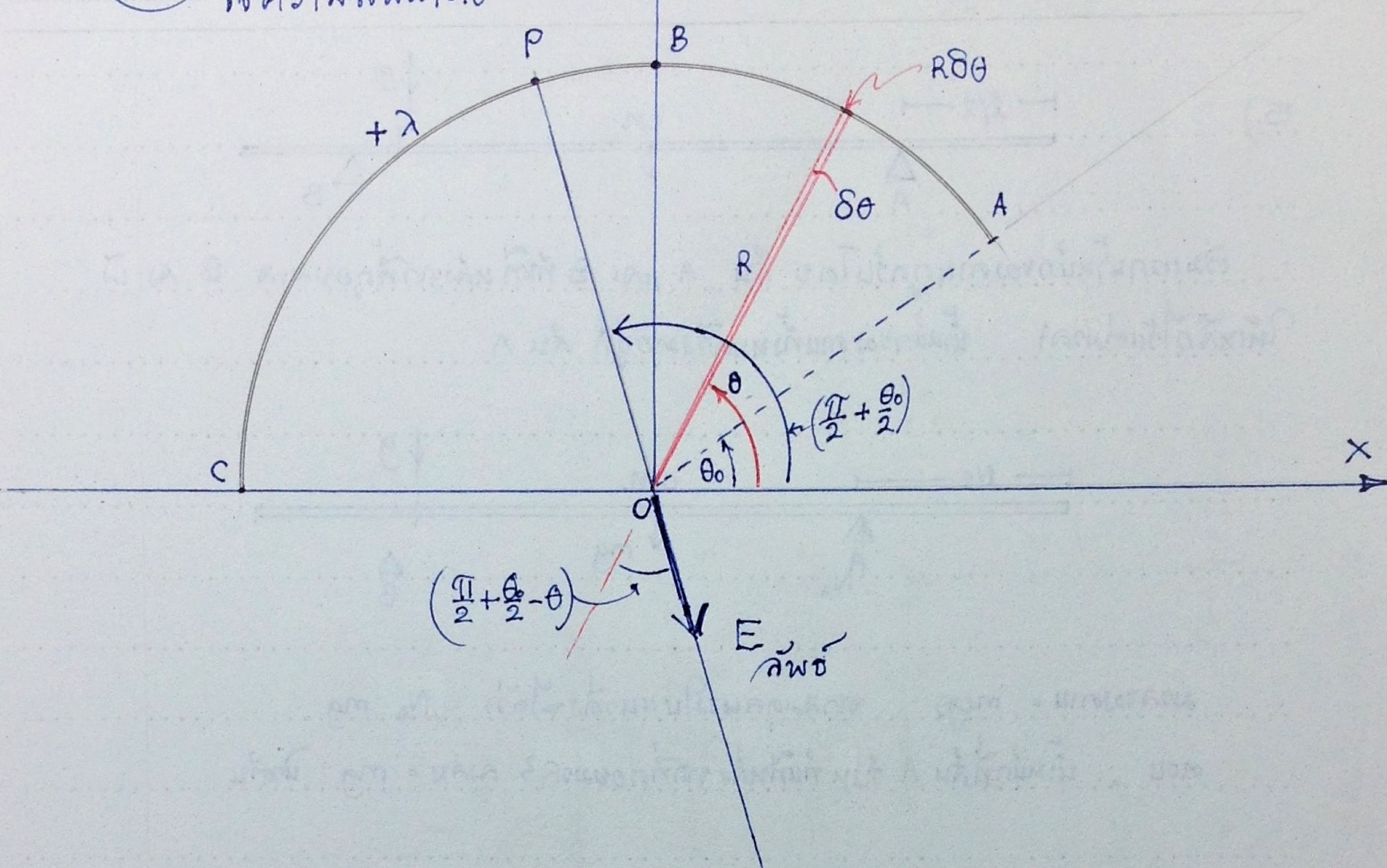
อ. 25.10.'65

การสอบปลายค่าย1 ที่ สสวท

(สอบ อา.16.10.'65)

ข้อ.3 คิดอีกแบบ

ใช้ความสมมาตร



เส้น PO แบ่งเส้นลวดอย่างสมมาตรเป็นสองซีกคือ PA กับ PC.

$$\delta E_{\text{ลัพธ์}} = \frac{\lambda R\delta\theta}{4\pi\varepsilon_0 R^2}\cos\left(\frac{\pi}{2}+\frac{\theta_0}{2}-\theta\right) = \frac{\lambda}{4\pi\varepsilon_0 R}\sin\left(\theta-\frac{\theta_0}{2}\right)\cdot\delta\theta$$

ขนาด $$E_{\text{ลัพธ์}} = \frac{\lambda}{4\pi\varepsilon_0 R}\int_{\theta=\theta_0}^{\pi}\sin\left(\theta-\frac{\theta_0}{2}\right)d\theta = \frac{-\lambda}{4\pi\varepsilon_0 R}\left[\cos\left(\theta-\frac{\theta_0}{2}\right)\right]_{\theta=\theta_0}^{\pi} = \frac{\lambda}{2\pi\varepsilon_0 R}\cos\frac{\theta_0}{2}$$

ตอบ (ค)

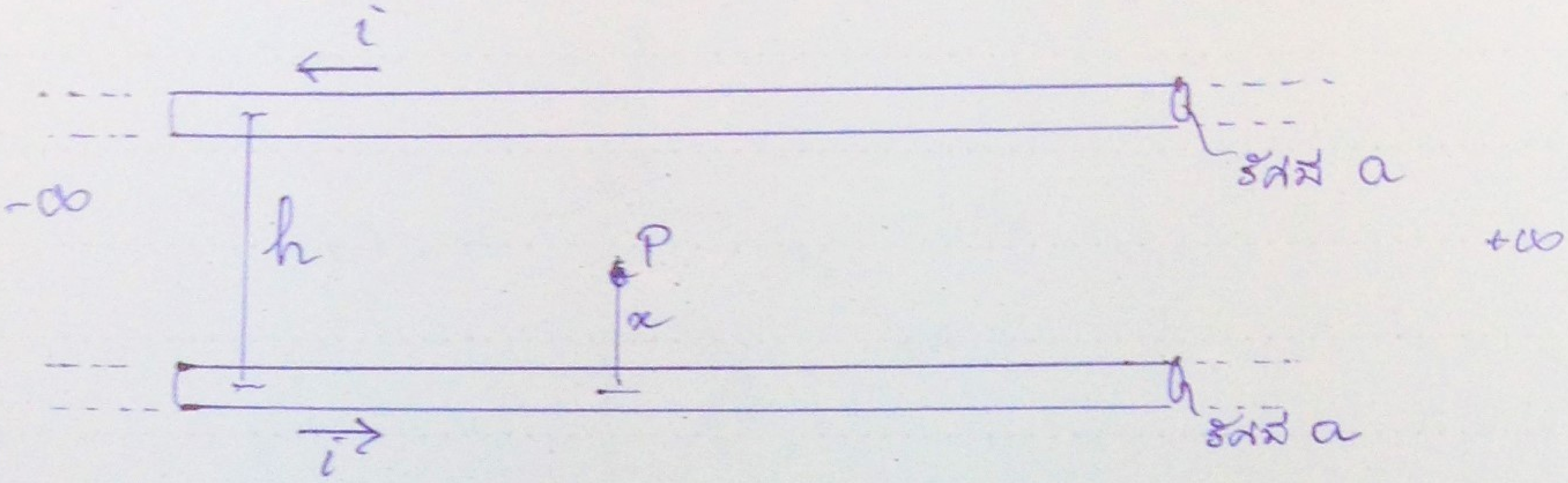
$$\therefore E_x = +\frac{\lambda}{2\pi\varepsilon_0 R}\cos\frac{\theta_0}{2}\sin\frac{\theta_0}{2} = \frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta_0$$ ตอบ (ก)

และ $$E_y = -\frac{\lambda}{2\pi\varepsilon_0 R}\cos\frac{\theta_0}{2}\cos\frac{\theta_0}{2} = -\frac{\lambda}{4\pi\varepsilon_0 R}(1+\cos\theta_0)$$ ตอบ (ข)

หมายเหตุ ~ คำตอบ (ค) อาจเขียนเป็น $\frac{\lambda}{2\pi\varepsilon_0 R}\cos\frac{\theta_0}{2} = \frac{\lambda\sqrt{2}}{4\pi\varepsilon_0 R}(1+\cos\theta_0)^{\frac{1}{2}}$

~ ไม่มีองค์ประกอบของ $\vec{E}_{\text{ลัพธ์}}$ ในแนวตั้งฉากกับ $\vec{PO}$

4.



(ก) Ampere's circuital law:

$$B_P \, 2\pi x = \mu_0 i \;, \quad B_P \, 2\pi(h-x) = \mu i \,,$$

จาก เส้นล่าง   จากเส้นบน

$$\therefore \underset{\text{ลัพธ์}}{B_P} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} + \frac{1}{h-x}\right) = \frac{\mu_0 i h}{2\pi x(h-x)}$$

(ข) Magnetic flux/metre

$$\Phi = \int_{x=a}^{h-a} B_P \, dx = \frac{\mu_0 i}{2\pi}\int_{x=a}^{h-a}\left\{\frac{1}{x} + \frac{1}{h-x}\right\} dx$$

$$= \frac{\mu_0 i}{2\pi}\left[\ln\left(\frac{h-a}{a}\right) - \ln\left(\frac{a}{h-a}\right)\right] = \frac{\mu_0 i}{\pi} \ln\left(\frac{h-a}{a}\right)$$

(ค) $L = \frac{\mu_0}{\pi} \ln\left(\frac{h-a}{a}\right)$

Note $\quad C = \frac{\pi \varepsilon_0}{\ln\left(\frac{h-a}{a}\right)} \;, \quad LC = \mu_0 \varepsilon_0$

Velocity of electrical wave $v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\therefore \; v = c$$